JOURNAL OF GUIDANCE, CONTROL, AND DYNAMICS
Vol. 23, No. 4, July–August 2000

Technical Comments

Brief discussion of previous investigations in the aerospace sciences and technical comments on papers published in the Journal of Propulsion and Power *are presented in this special department. Entries must be restricted to a maximum of 1000 words, or the equivalent of one* Journal *page including formulas and figures. A discussion will be published as quickly as possible after receipt of the manuscript. Neither the AIAA nor its editors are responsible for the opinions expressed by the correspondents. Authors will be invited to reply promptly.*

Comment on "Does the Phugoid Frequency Depend on Speed?"

N. Ananthkrishnan*
Indian Institute of Technology, Bombay, Mumbai 400 076, India

I. Introduction

IN a recent Note with a provocative title, the authors[1] questioned the popular notion that the phugoid frequency varies inversely with speed. They cited data from the literature that show an increase in phugoid frequency with increasing speed in some instances. Based on an analytical expression for the phugoid frequency, they conclude that the phugoid frequency is independent of speed provided the nondimensional aerodynamic and thrust derivatives are constant with respect to variations in speed. In particular, their conclusion implies that any observed inverse variation of the phugoid frequency with speed is purely due to the behavior of the nondimensional aerodynamic and thrust derivatives.

In the present Note, we point out that the conclusions in Ref. 1 are not entirely correct. Much of the confusion in Ref. 1 is seen to be due to an incomplete statement of the problem at hand. Once the question is correctly posed, the conditions under which the phugoid frequency varies with speed become obvious.

II. Problem Statement

Consider an airplane flying steady and level with speed U_e at an altitude H_e (density ρ_e). Balance of forces in the vertical direction gives $\frac{1}{2}\rho_e U_e^2 S C_{L_e} = W$, where S is the wing area, W is the airplane weight, and C_{L_e} is the lift coefficient at that flight condition. The airplane response to a small perturbation about this equilibrium state can be described in terms of the phugoid (or long-period) and the short-period modes. When these two modes are oscillatory, as is usually the case, the small perturbation dynamics can be specified by stating the frequency and damping of these modes.

Now, the question posed is as follows: "If the equilibrium state of the airplane is changed from one steady, level flight (ρ_1, U_1, C_{L_1}) to another, (ρ_2, U_2, C_{L_2}), what is the corresponding change in the phugoid frequency?"

It is important to recognize that the two equilibrium states are related by the following constraint:

$$\tfrac{1}{2}\rho_1 U_1^2 S C_{L_1} = \tfrac{1}{2}\rho_2 U_2^2 S C_{L_2} = W \tag{1}$$

Thus, as speed varies from U_1 to U_2, both ρ and C_L will change, in general, subject to the constraint in Eq. (1). In any case, both ρ and C_L cannot remain fixed as U is varied. It is common to consider two special cases: 1) variation of U with C_L kept constant, that is, $C_{L_1} = C_{L_2}$ and 2) variation of U with altitude H kept constant, that is $\rho_1 = \rho_2$.

It is apparent from the data in Table 1 of Ref. 1 that the authors have considered only case 2 in their Note, although this has not been stated explicitly.

Received 9 April 1999; accepted for publication 4 December 1999.

*Assistant Professor, Department of Aerospace Engineering; akn@aero.iitb.ernet.in. Member AIAA.

III. Analysis

With the problem stated clearly, it is now easy to see how the frequency of the phugoid mode varies with variation in speed in the two cases just listed. For this purpose, it is adequate to use the standard approximation for phugoid frequency available in the literature (for example see Roskam[2]), although the same conclusions can be arrived at by using the more complicated expression in Ref. 1.

The phugoid frequency is usually approximated as follows, where $\bar{q}_1 = \frac{1}{2}\rho_1 U_1^2$:

$$\omega_p = \sqrt{\frac{-Z_u g}{U_1}} = \sqrt{\frac{\bar{q}_1 S\left(C_{L_u} + 2C_{L_1}\right)}{mU_1}\frac{g}{U_1}} \tag{2}$$

Case 1

When C_{L_1} is constant, we can write $\bar{q}_1 S = W / C_{L_1}$ in Eq. (2), and a little manipulation yields the result

$$\omega_p = (g/U_1)\sqrt{C_{L_u}/C_{L_1} + 2} \tag{3}$$

which, when $C_{L_u}/C_{L_1} \ll 2$, gives the Lanchester result, $\omega_p = g\sqrt{(2)}/U_1$. Thus, Eq. (3) clearly indicates that the inverse variation of the phugoid frequency with speed is no conjecture, but a fact, in the case where C_L is maintained constant as the speed is varied, provided the aerodynamic and thrust derivatives are assumed to be constant with speed. It is not difficult to show that, using the more complicated expression for the phugoid frequency quoted in Ref. 1, manipulations similar to earlier yield

$$\omega_p = \sqrt{\frac{g^2\left(C_{L_u}/C_{L_1} + 2\right) + k_1}{U_1^2 + k_2}} \tag{4}$$

where k_1 and k_2 are constants, provided the aerodynamic and thrust derivatives do not vary with speed, and usually $k_2 \ll U_1^2$. Once again, Eq. (4) suggests an inverse variation of the phugoid frequency with speed.

Case 2

When ρ_1 is consant, we can replace $\bar{q}_1$ in Eq. (2) with $\bar{q}_1 = \frac{1}{2}\rho_1 U_1^2$, where from the following expression for the phugoid frequency can be derived:

$$\omega_p = g\sqrt{\frac{C_{L_u}\rho_1 S}{2W} + \frac{2}{U_1^2}} \tag{5}$$

In the case where the first term under the root in Eq. (5) is much smaller than the second term, Eq. (5) reduces to the Lanchester formula for the phugoid frequency. On the other hand, when the first

Table 1 Variation of phugoid frequency in case 2[a]

Mach number	ω_p, rad/s
a) Lockheed Jetstar	
0.2	0.193
0.4	0.082
b) Convair CV-880M	
0.2	0.149
0.25	0.145

[a]At sea level.

term dominates, the phugoid frequency works out to be approximately independent of speed. It is possible to draw similar conclusions with the more complicated expression for phugoid frequency used in Ref. 1.

We now seek data from the literature in support of the conclusions in case 2. In practice, the nondimensional aerodynamic and thrust derivatives vary with speed, especially in the transonic regime. We, therefore, look for data at low Mach numbers, where the aerodynamic and thrust derivatives may be expected to be approximately constant. A perusal of the data in Ref. 3 provides several examples, one each of which illustrates the two conclusions arrived at earlier for case 2 and are cited in Table 1. In case of airplane a, the phugoid frequency shows a decrease with increase in speed, whereas for airplane b, the phugoid frequency is nearly constant with increase in speed.

IV. Conclusions

It is shown that the variation of the phugoid frequency with speed depends on the manner in which the speed is varied between two level-flight equilibrium states. If C_L is kept fixed as the speed is varied, the phugoid frequency shows the well-known inverse dependence on speed. It is, therefore, incorrect to suggest that the inverse variation of the phugoid frequency with speed is only due to changes in the aerodynamic and thrust derivatives. However, the phugoid frequency does turn out to be independent of speed in certain cases, when the speed is varied with the altitude kept constant.

References

[1] Pradeep, S., and Kamesh, S., "Does the Phugoid Frequency Depend on Speed?" *Journal of Guidance, Control, and Dynamics*, Vol. 22, No. 2, 1999, pp. 372, 373.

[2] Roskam, J., *Airplane Flight Dynamics and Automatic Flight Controls*, Vol. 1, Roskam Aviation and Engineering Corp., Lawrence, KS, 1979, p. 6:34.

[3] Schmidt, L. V., *Introduction to Aircraft Flight Dynamics*, AIAA, Reston, VA, 1998, pp. 333–354.

Reply by the Author to N. Ananthkrishnan

S. Pradeep*
Indian Institute of Science, Bangalore 560012, India

Introduction

IN a recent paper[1], contrary to popular belief, it was shown that the phugoid frequency is independent of the forward speed. This was done by using a new approximation to the phugoid frequency that is very accurate. The research suggested that it was not speed, but the conspired variation of the aerodynamic derivatives with speed that caused the decrease of phugoid frequency with an increase in speed. To support the argument, counterexamples in which the phugoid frequency increased with an increase in speed were shown. This broke a century-old belief and has, quite naturally, stirred up a debate. In a recent note,[2] it is claimed that the results quoted are only partially correct. The misunderstanding has cropped up because the approximation to the phugoid frequency that is used to arrive at the conclusion in the first study is inadequate and requires amelioration. The purpose of this note is to clarify the issues raised.

Clarification

The controversial result[1] is that if the speed alone is varied, keeping the aerodynamic and the thrust derivatives fixed by some means, the phugoid frequency remains unchanged, establishing that the phugoid frequency is independent of speed.

Table 1 Percentage error in the 2DOF phugoid frequency approximation from the exact value

Aircraft	Flight phase	% error
A		−15.02
B	1	−14.38
	2	−17.93
	3	−7.41
C	1	−24.27
	2	−27.52
	3	−19.78
D	1	−12.63
	2	10.50
	3	19.66
E	1	−17.92
	3	33.10
F	1	−17.41
	2	−34.53
	3	−6.53

Ananthkrishnan[2] rephrases the problem in the following manner: "If the equilibrium state of the airplane is changed from one steady, level flight (p_1, U_1, C_{1a}) to another (p_2, U_2, C_{2a}) what is the corresponding change in phugoid frequency?" The two equilibrium states are shown to be related by the following constraint.

$$\tfrac{1}{2}p_1U_1^2SC_{1a} = \tfrac{1}{2}p_2U_2^2SC_{ta} \tag{1}$$

As the speed is varied from U_1 to U_2, both the density and the coefficient of lift change subject to the constraint in Eq. (1). On the basis of this, two special cases, one with the coefficient of lift kept unchanged and the other with the altitude kept unchanged, are examined. Until this point, the argument is flawless. The 2DOF approximation[3] is then used to affirm that the phugoid frequency is inversely proportional to speed.[2] For a study of this nature, the 2DOF approximation, $w_p = gZ_u/U_1$ is inappropraite because it frequently results in large errors. This is the source of the incorrect inferences of Ref. 2.

To validate this claim, Table 1 presents the percentage error in the 2DOF phugoid frequency approximation from the exact value, calculated for the same set of data that was used in former studies by the author.[1] It is taken from Appendix C of Roskam's text on flight dynamics.[4] This collection of data pertains to six modern aircraft in a total of 16 flight conditions. The chosen aircraft possess dissimilar missions: a small four-place transportation airplane, a 19-passenger commuter airliner, a small jet trainer, a medium-sized high performance business jet, a supersonic fighter-bomber, and a large wide-body jet transport. The flight conditions extend from power approach at sea level to cruise at medium and high altitudes. The database is thus representative of a whole gamut of airplanes and flight conditions. In the 15 cases examined, the percentage error between the exact value of the phugoid frequency and that calculated from the approximation varies from 6.53% to 33.10%. It is not surprising that judgments based on this expression are erroneous.

On the contrary, the approximation[3] to the phugoid frequency

$$w_p = \sqrt{\frac{g(M_\alpha Z_u - M_\alpha Z_a)}{M_g Z_a - U_1 M_\alpha}} \tag{2}$$

does not differ from the exact value by more than 4% in the 15 cases considered. Conclusions based on Eq. (2) have, therefore, greater authenticity. Reference 2 also states that similar conclusions can be reached from the nondimensional version of Eq. (2). The nondimensional version derived in Ref. 2 [Eq. (4) of Ref. 2] is incorrect, because U_1 has not been canceled with q_1. The correct expression is[1]

$$w_p = \sqrt{\frac{g\left\{C_{m_0}\left(C_{L_0} + 2C_{L_1}\right) - \left(C_{w_0} + 2C_m\right)\left(C_{l_0} + C_{D_1}\right)\right\}}{\left(C_{m_q}\bar{c}/2\right)\left(C_{L_0} + C_{D_1}\right) + (2m/pS)C_{m_\alpha}}} \tag{3}$$

Received 15 May 2000; accepted for publication 14 June 2000.

*Associate Professor, Department of Aerospace Engineering. Member AIAA.